SOLUTIONS

2024-Mock JEE Advanced-3 (CBT) | Paper-1

PART-I	PHYSICS

SECTION-1

1.(D) $1\,MSD = \frac{l}{\cos\theta}$

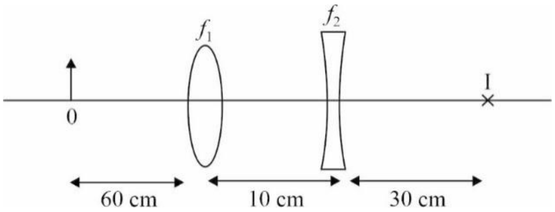
$1VSD = l$

$LC = 1\,MSD - 1VSD$

$= l\left(\frac{1}{\cos\theta} - 1\right) = l\frac{(1-\cos\theta)}{\cos\theta} \approx \frac{l\,\theta^2}{2} = 50\,mm \times \frac{1}{2} \times \left(\frac{3.6}{180}\pi\right)^2 = 0.1\,mm$

2.(B) $C = C_v + \frac{RT}{V}\frac{dV}{dT}$; $R = \frac{3R}{2} + \frac{RT}{V}\frac{dV}{dT}$; $\frac{dV}{V} + \frac{1}{2}\frac{dT}{T} = 0 \Rightarrow VT^{1/2} =$ constant

3.(D)

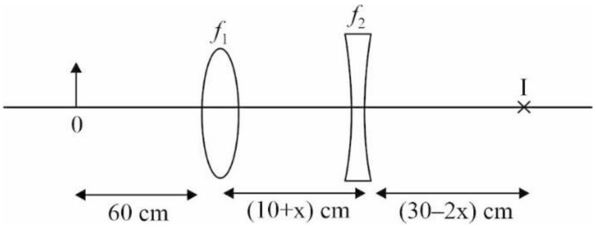


From Lens-maker's formula, $f_1 = 15\,cm$ and $f_2 = -15\,cm$

$L_1 : \frac{1}{v_1} + \frac{1}{60} = \frac{1}{15} \quad \Rightarrow \quad v_1 = 20\,cm$

$L_2 : u_2 = 10\,cm$

$\frac{1}{v_2} - \frac{1}{10} = \frac{1}{-15} \quad \Rightarrow \quad \frac{1}{v_2} = \frac{1}{10} - \frac{1}{15} = \frac{3.2}{30} = \frac{1}{30} \quad \Rightarrow \quad v_2 = 30\,cm$



$L_1 :\ u = -60\,cm,\ v_1 = 20\,cm$

$L_2 :\ u_2 = 20 - (10 + x) = 10 - x$

$v_2 = 30 - 2x$

$\frac{1}{30-2x} - \frac{1}{10-x} = -\frac{1}{15} \quad \Rightarrow \quad \frac{10 - x - 30 + 2x}{300 - 50x + 2x^2} = -\frac{1}{15}$

$\Rightarrow \quad 300 - 15x = 300 - 50x + 2x^2 \quad \Rightarrow \quad 2x^2 - 35x = 0$

$\Rightarrow \quad x = 0, \frac{35}{2} \quad \therefore \quad x = 17.5\,cm$

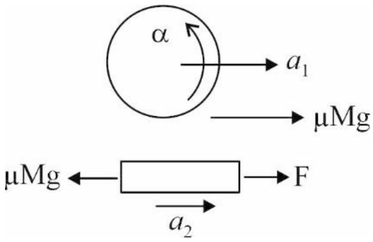
4.(C) $\frac{dN}{dt} = R - \lambda N \quad \Rightarrow \quad \int_{No}^{N} \frac{dN}{R - \lambda N} = \int_{o}^{t} dt \quad \Rightarrow \quad N(t) = \frac{R}{\lambda} - \left(\frac{R}{\lambda} - No\right)e^{-\lambda t}$

After one. Half life, $e^{-\lambda t} = \frac{1}{2}$; $N = \frac{R}{\lambda} - \left(\frac{R}{\lambda} - No\right) \times \frac{1}{2} = \frac{R}{2\lambda} + \frac{No}{2}$

SECTION-2

5.(BC) $\mu Mg = Ma_1$

$\Rightarrow \quad a_1 = \mu g \quad \ldots(i)$



$F - \mu Mg = Ma_2$

$\Rightarrow \quad a_2 = \frac{F}{M} - \mu g \quad \ldots(ii)$

$\mu MgR = \frac{2}{5} MR^2 \alpha$

$\Rightarrow \quad \alpha = 2.5 \frac{\mu g}{R} \quad \ldots(iii)$

For no slipping, $a_1 + R\alpha = a_2$

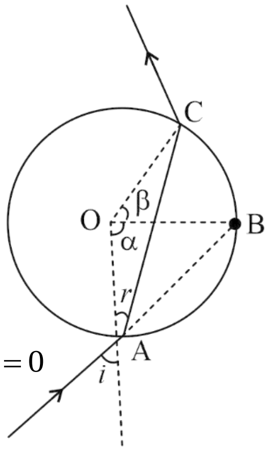
$\mu g + 2.5\mu g = \frac{F}{M} - \mu g$

$\Rightarrow \quad F = 4.5\mu Mg \quad \ldots(iv) \qquad \therefore \quad a_2 = 3.5\mu g \quad \ldots(v)$

$F = kt = 4.5\mu Mg$; $t = 4.5 \frac{\mu Mg}{k}$

6.(BC) $\alpha = \pi - 2i$

$\alpha + \beta = \pi - 2r \qquad \therefore \qquad \beta = 2i - 2r$



Given, $\alpha = \beta$

$\pi - 2i = 2i - 2r \qquad \Rightarrow \qquad r = 2i - \frac{\pi}{2}$

Snell's Law: $1 \times \sin i = \sqrt{6} \sin\left(2i - \frac{\pi}{2}\right)$

$\Rightarrow \quad \sin i = \sqrt{6}\left(2\sin^2 i - 1\right) \qquad \Rightarrow \qquad 2\sqrt{6}\sin^2 i - \sin i - \sqrt{6} = 0$

$\sin i = \frac{1 + \sqrt{1+48}}{4\sqrt{6}} \qquad = \frac{8}{4\sqrt{6}} = \sqrt{\frac{2}{3}}$

$\sin r = 2\sin^2 i - 1 \qquad = \frac{4}{3} - 1 = \frac{1}{3}$

7.(BC) $\omega = \frac{d\theta}{dt}$; $\qquad \Omega = \frac{d}{dt}(2\theta) = 2\omega$

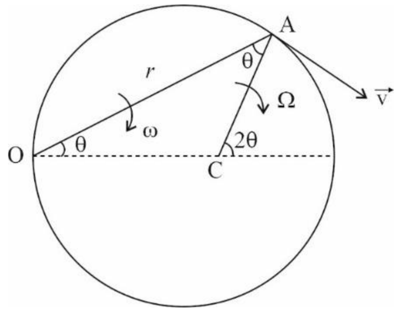
$r = 2R\cos\theta \quad \therefore \quad V = R\Omega = 2R\omega$

$\therefore \quad L = mV_{\perp} r = mV \cos\theta r$

$= m \times 2R\omega \times \cos\theta \times 2R\cos\theta$

$= 4m\,\omega R^2 \cos^2\theta$

$= 4\,m\,\omega R^2 \times \frac{3}{4} = 3m\omega R^2$



$$\therefore \qquad \tau = \frac{dL}{dt} = \frac{d}{dt}\left(4m\omega R^2 \cos^2\theta\right)$$

$$= 4m\omega R^2 \frac{d}{dt}\left(\cos^2\theta\right)$$

$$= -4m\omega^2 R^2 \sin^2\theta$$

$$|\tau| = 4m\omega^2 R^2 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}m\omega^2 R^2$$

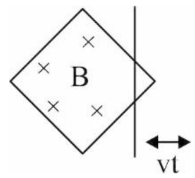
8.(BC) $n = 4 \xrightarrow{E_1} n = 2;\ n = 2 \xrightarrow{E_2} n = 1$

$$E_1 = 13.6eV\left\{\frac{1}{2^2} - \frac{1}{4^2}\right\} = \frac{3}{16}\times 13.6eV$$

$$E_2 = 13.6eV\left\{\frac{1}{1^2} - \frac{1}{2^2}\right\} = \frac{3}{4}\times 13.6eV; \qquad \lambda = \frac{hc}{E};\quad P = \frac{E}{C}$$

$$\therefore \qquad \alpha = \frac{\lambda_1}{\lambda_2} = \frac{E_2}{E_1} = 4; \qquad \beta = \frac{E_2}{E_1} = \frac{1}{4}; \qquad \gamma = \frac{1}{4}$$

9.(BC) Case-1: $O \le t \le \dfrac{l}{\sqrt{2}V}$

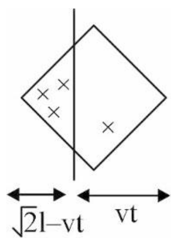


$$A = l^2 - 2v^2t^2;\ \phi = BA; \qquad |\varepsilon| = \left|\frac{d\phi}{dt}\right| = 2Bv^2t$$

$$I = \frac{2Bv^2t}{R}; \qquad P = \frac{d\theta}{dt} = I^2R = \frac{4B^2V^4}{R}t^2$$

$$Q_1 = \frac{4B^2V^4}{R}\int_{.0}^{l/\sqrt{2}v} t^2dt = \frac{4B^2V^4}{R}\times\frac{1}{3}\times\frac{l^3}{2\sqrt{2}V^3} = \frac{\sqrt{2}}{3}\frac{B^2Vl^3}{R}$$

Case 2: $\dfrac{l}{\sqrt{2}v} \le t < \sqrt{2}\dfrac{l}{v}$



$$A = \left(\sqrt{2}l - vt\right)^2; \qquad \phi = BA$$

$$|\varepsilon| = \left|\frac{d\theta}{dt}\right| = 2BV\left(\sqrt{2}l - vt\right)$$

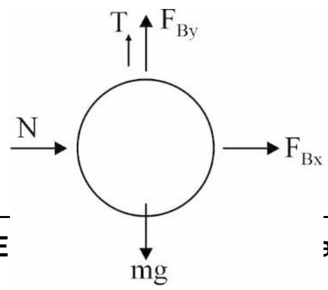
$$I = \frac{2BV}{R}\left(\sqrt{2}l - vt\right); \quad \frac{d\theta}{dt} = I^2R$$

$$\therefore \qquad Q_2 = \frac{4B^2V^2}{R}\int_{l/\sqrt{2}v}^{\sqrt{2}l/v}\left(\sqrt{2}l - vt\right)^2 = \frac{\sqrt{2}B^2vl^3}{3R} \qquad \therefore \qquad Q_{Total} = \frac{2\sqrt{2}B^2vl^3}{3R}$$

(C) Charge flown, $\Delta Q = \dfrac{\Delta\phi}{R} = \dfrac{Bl^2}{R}$

10.(CD) Since the sphere is a perfect sphere, we can assume that the sphere is entirely surrounded by the liquid. Mass of sphere, $M = \dfrac{4}{3}\pi R^3 \times 2\rho = \dfrac{8}{3}\pi R^3\rho$

Mass of liquid displaced, $M' = \dfrac{M}{2}$



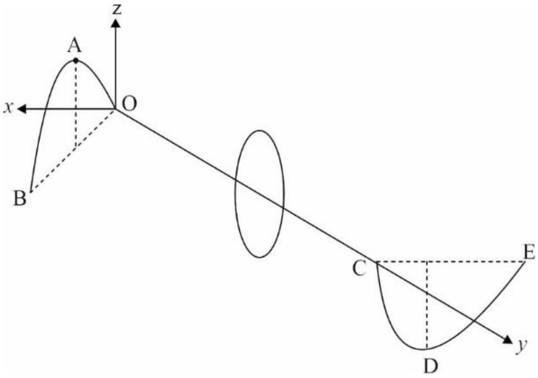
$\therefore \quad F_{By} = M'g = \frac{M}{2}g \quad \text{and} \quad F_{Bx} = M'a_0 = \frac{M}{2}a_0$

$\therefore \quad N = \frac{M}{2}a_0 = \frac{4}{3}\pi R^3 \rho a_0 \quad ; \quad T = \frac{M}{2}g = \frac{4}{3}\pi R^3 \rho g$

$F_B, net = \sqrt{F_{Bx}^2 + F_{By}^2} = \frac{M}{2}\sqrt{a_0^2 + g^2} = \frac{4}{3}\pi R^3 \rho \sqrt{a_0^2 + g^2}$

SECTION-3

1-2. **1.(320) - 2.(100)**



$T = \frac{2u_z}{g} = 2 \times \frac{u}{2g} = \frac{u}{g}$

Object : $R_x = u_x \times T = u \times \frac{u}{g} = \frac{u^2}{g} = f$

$u_y \times T = 2f$

$H = \frac{u_z^2}{2g} = \frac{u^2}{4 \times 2g} = \frac{f}{8}; \qquad A = \left(\frac{f}{2}, f, \frac{f}{8}\right)$

At O: Lens formula: $\frac{1}{v} + \frac{1}{4f} = \frac{1}{f} \quad \Rightarrow \quad v = \frac{4f}{3}$

$\therefore \quad c\left(0, 4f + \frac{4}{3}f, 0\right) = C\left(0, \frac{16}{3}f, 0\right) \quad \therefore \quad x_1 + 3y_1 + z_1 = 16f = 320cm$

At A $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}; \qquad \frac{1}{v} + \frac{1}{3f} = \frac{1}{f} \quad \Rightarrow \quad v = \frac{3}{2}f$

$m = \frac{v}{u} = \frac{\frac{3}{2}f}{-3f} = -\frac{1}{2} \quad \therefore\ m = \frac{\Delta x_I}{\Delta x_0} = -\frac{1}{2} \quad \Rightarrow \quad \Delta x_1 = -\frac{1}{2} \times \frac{f}{2} = -\frac{f}{4}$

Also, $m = \frac{\Delta z_i}{\Delta z_0} = -\frac{1}{2} \quad \Rightarrow \quad \Delta z_1 = -\frac{1}{2} \times \frac{1}{8} = -\frac{f}{16}$

$D\left(-\frac{f}{4}, 4f + \frac{3}{2}f, -\frac{f}{16}\right) = D\left(-\frac{f}{4}, \frac{11}{2}f, -\frac{f}{16}\right)$

$\therefore \quad x_2 + y_2 + 4z_2 = -\frac{f}{4} + \frac{11}{2}f - \frac{f}{4} = 5f = 100cm$

3.(16) $\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_2}{r_1 + r_2} = \frac{2 \times 3 + 1 \times 2}{5} = \frac{8}{5}$

$Q = CV$

$Qi = 10 \times \frac{8}{5} = 16$

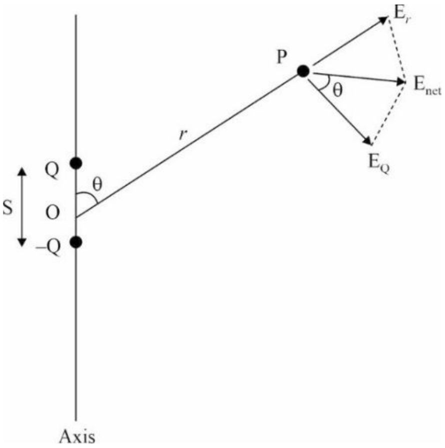
4.(184) Discharging: $q = Q_0 e^{-t/z}$, $\tau = RC = 10ns$

$\Rightarrow \quad 1.6 \times 10^{-19} = 16 \times 10^{-12}\, e^{-t/\tau} \quad \Rightarrow \quad 10^{-8} = e^{-t/\tau}$

$\Rightarrow \quad \tau \ln\left(10^8\right) = t \quad \Rightarrow \quad t = 10ns \times 8 \times 2.3 = 184ns$

5-6. **5.(2) - 6.(2)**

$\vec{E}$ due to a uniformly charged sphere is same as the entire charge is placed at its centre.



The two charges separated by a small distance form a dipole of dipole moment, $P = Q \times S = \rho \times \frac{4}{3}\pi R^3 \times S$

$E_r = \frac{2Kp\cos\theta}{r^3} \quad ; \quad E_Q = \frac{Kp\sin\theta}{r^3}$

$\tan\theta = \frac{E_r}{E_\theta} = 2\cot\theta \quad \Rightarrow \quad \tan^2\theta = 2$

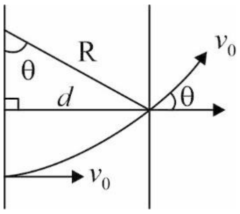
$E_{net} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta} = \sqrt{2} \times \frac{1}{4\pi \in_0} \times \rho \times \frac{4}{3}\pi R^3 \times S \times \frac{1}{r^3} = \sqrt{2}\,\frac{\rho S R^3}{3\varepsilon_0 r^3}$

Hence, $\beta = 2$

SECTION-4

7.(30) $\sin\theta = \frac{d}{R}; R = \frac{mv}{qB} \quad \therefore \quad \sin\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 30°$



8.(10) Distance moved by centre of A in Reaching some point from where it started $= 2\pi(4R)$. Distance center should move for one rotation $= 2\pi R$

No. of Rotation $a = \frac{2\pi 4R}{2\pi R} = 4$

Similarly No. of Rotation $c = \frac{2\pi(2R)}{2\pi R} = 2$; $(2a + c) = 2 \times 4 + 2 = 10$

9.(82) $T = T_s + (T_0 - T_s)e^{-kt}$; $T = \frac{T_0}{2} + \frac{T_0}{2}e^{-kt}$

$t = to,\ \ 0.9\,T_0 = \frac{T_0}{2} + \frac{T_0}{2}e^{-kt_0} \Rightarrow e^{-kt_0} = 0.8$

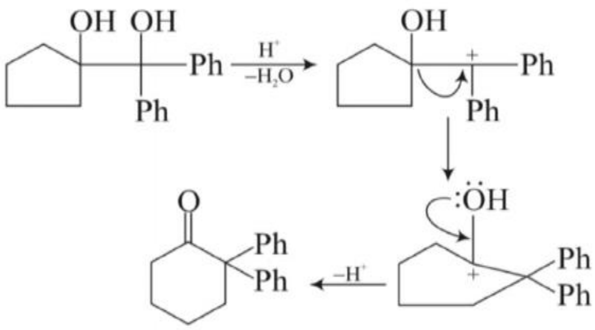
$t = 2t_0,\ \ T = \frac{T_0}{2} + \frac{T_0}{2}e^{-2kt_0} = \frac{T_0}{2} + \frac{T_0}{2}(0.8)^2$

$= (0.5 + 0.32)T_0 = 0.82\,T_0$; 82%

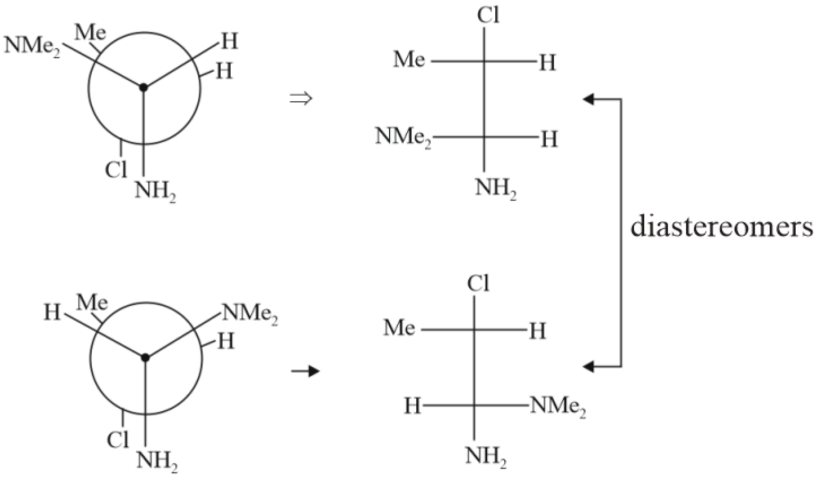
PART-II	CHEMISTRY

SECTION-1

1.(A)



2.(D)



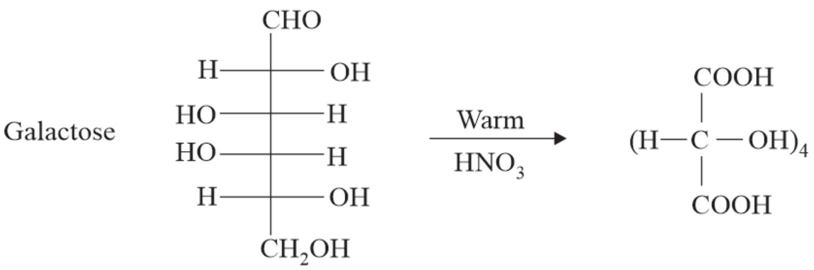
3.(B) Packing fraction $= \dfrac{3\times \pi r^2}{\dfrac{\sqrt{3}}{4}a^2 \times 6} = 0.9069 \approx 0.91$

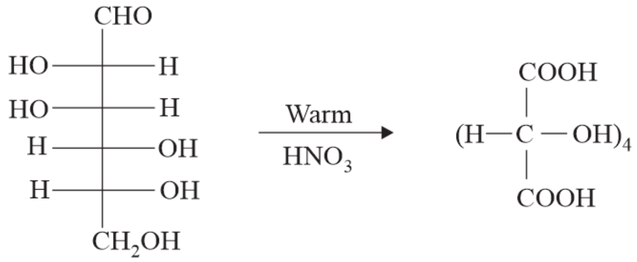
$a = 2r \quad \Rightarrow \quad Z = \dfrac{1}{3}\times 6 + 1 = 3$

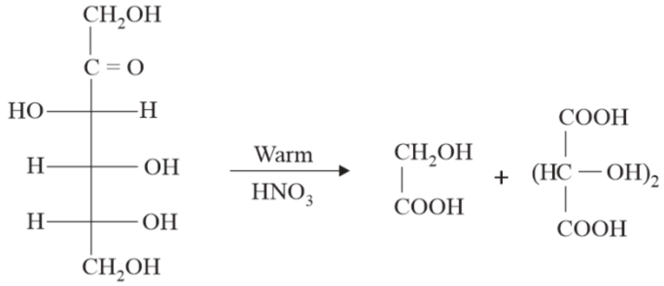
4.(D) $[Cr(H_2O)_6]^{3+}$ contains 3 unpaired electrons whereas $[Zn(H_2O)_6]^{2+}$ and $[Fe(CN)_6]^{4-}$ contains zero unpaired electron. $[Mn(H_2O)_6]^{2+}$ has 5 unpaired electrons. More the number of unpaired electrons more is the magnetic moment.

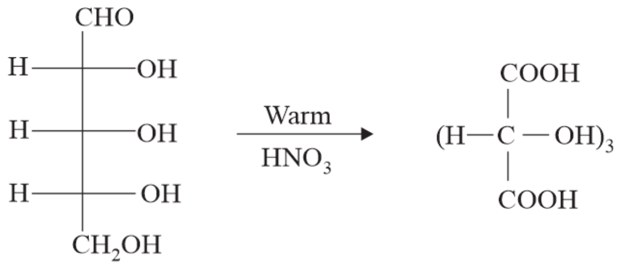
SECTION-2

5.(AC)

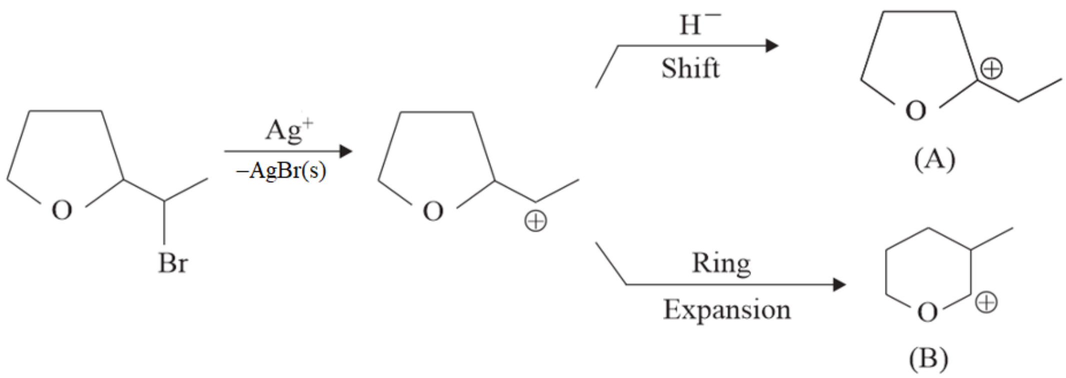


Mannose 

Fructose 

Ribose 

6.(BC)



(A) and (B) undergo nucleophilic attack by H_2O giving the desired products.

7.(ABC)

8.(D) $\underset{(2\,atm)}{A} \xrightarrow[\Delta]{\text{Isochoric}} \underset{(5\,atm)}{B}$ P increases but V is constant $C \xrightarrow[\text{Compression}]{\text{Isothermal reversible}} A$

9.(ABCD)

$$Ni + 4CO \xrightarrow{50°C-60°C} [Ni(CO)_4] \xrightarrow{200°C-230°C} Ni + 4CO\uparrow$$

10.(B) $\underset{(A)}{K_2Cr_2O_7} \xrightarrow[+H_2SO_4]{NH_4Cl} \underset{(C)}{CrO_2Cl_2} \xrightarrow[AgNO_3]{NaOH} \underset{(D)}{Ag_2CrO_4} \xrightarrow[\text{Solution}]{NH_3} \text{Yellow solution}$

SECTION-3

1.(56.7) $\% = \frac{70}{100} \times \frac{90}{100} \times \frac{90}{100} \times 100 = 56.7$

2.(20.25)

$$\% = \frac{25}{100} \times \frac{90}{100} \times \frac{90}{100} \times 100 = 20.25$$

3.(4.94) $\Delta S_{vap} = \frac{\Delta H_{vap}}{T} = \frac{540 \times 18}{373\,(K)} \text{Cal / mol}$

$$\Delta S_{fus} = \frac{\Delta H_{fus}}{T} = \frac{1440}{273} \text{ Cal / mol} ; \qquad \frac{\Delta S_{vap}}{\Delta S_{fus}} \approx 4.94$$

4.(508.33)

$\Delta G = \Delta H - T\Delta S \qquad \Delta G = 0$ for reaction in equilibrium

$$T = \frac{\Delta H}{\Delta S} = \frac{30500}{60} = 508.33\text{ K}$$

5.(175.79)

Moles given by NaCl = $2 \times \frac{3.5}{58.5}$

Moles given by $MgCl_2$ = $3 \times \frac{1.5}{95}$

Total moles = $2 \times \frac{3.5}{58.5} + 3 \times \frac{1.5}{95} = 0.167$

Mass of water = 100 – 3.5 – 1.5 = 95

Molality (m) $= \frac{0.167}{95} \times 1000 = 1.75789 = 175.789$

6.(100.89)

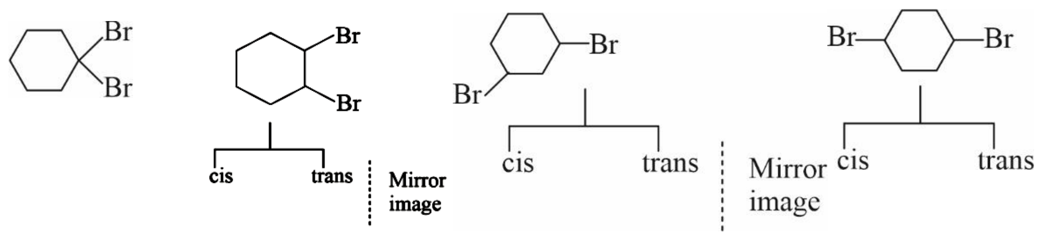
$$\Delta T_b = K_b . m$$

$$T_b - T_b^0 = 0.51 \times 175.79 \times 10^{-2}$$

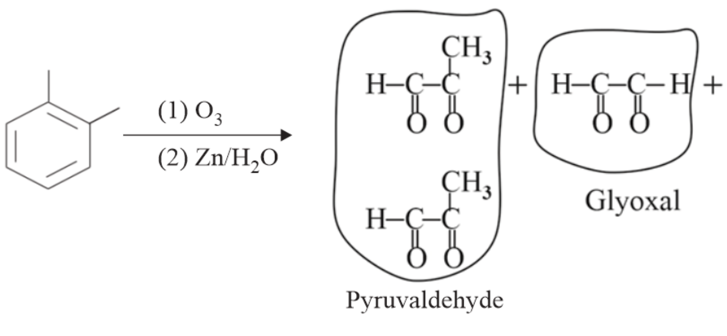
$$T_b = 100 + 0.8976 = 100.89°C$$

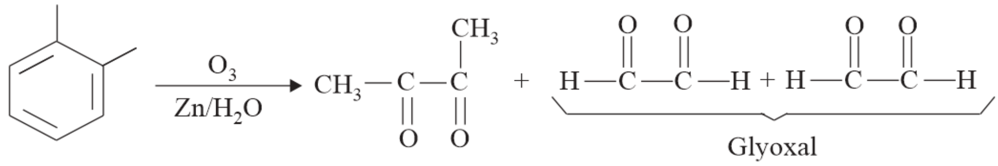
SECTION-4

7.(9)



8.(5)





Glyoxal : Pyruvaldehyde
3 : 2

9.(6) Example of $[Ma_2b_2c_2]$

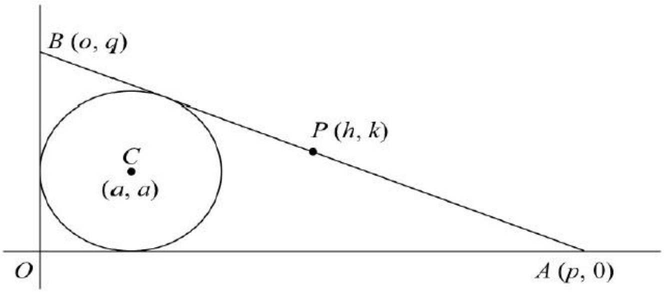
aa	aa	ab	ab	Mirror	aa
bb	bc	ab	bc	image	ac
cc	bc	cc	ac		bb

5 are geometrical isomers and 1 among them is optically active, hence with its mirror image, total is six.

PART-III	MATHEMATICS

SECTION-1

1.(D) The given circle has its centre at $C(a, a)$ and its radius is a so that it touches both the axes along which lies the two sides of the triangle. Let third side be $\frac{x}{p}+\frac{y}{q}=1.$



So that A is (p, 0) and B is (0, q) and the line AB touches the given circle. Since $\angle AOB$ is a right angle, AB is a diameter of the circumcircle of the triangle, AOB. So the circumcentre $P(h, k)$ of the triangle AOB is the mid-point of AB.

i.e. $2h = p, 2k = q$

Now equation of AB is $\frac{x}{p}+\frac{y}{q}=1,$ which touches the given circle

$$\therefore \qquad \frac{a(p+q)-pq}{\sqrt{p^2+q^2}}=a$$

$$\Rightarrow \qquad a^2(p+q)^2+p^2q^2-2apq(p+q)=a^2(p^2+q^2)$$

$$\Rightarrow \qquad 2a^2-2a(p+q)+pq=0$$

$\Rightarrow \qquad 2a^2-2a(2h+2k)+2h\cdot 2k=0.$ Hence the locus of $P(h, k)$ is $a^2-2a(x+y)+2xy=0.$

Since it passes through $(2,-1)$

$$a^2-2a=2\times 2\times 1=4$$

2.(A) The given curve intersects the x–axis at points where $\sin(\beta x)=0.$ That is $x_n=\frac{n\pi}{\beta}, n=0,1,2,$

The given function is positive in $(x_{2k,}x_{2k+1})$ and negative in $(x_{2k+1,}x_{2k+2}).$

Area $S_n=\left|\int_{\frac{n\pi}{\beta}}^{\frac{(n+1)\pi}{\beta}} y dx\right| = (-1)^n \int_{\frac{n\pi}{\beta}}^{\frac{(n+1)\pi}{\beta}} e^{-\alpha x}\sin(\beta x)dx$

But we know that:

$$\int e^{-\alpha x}\sin(\beta x)=-\frac{e^{-\alpha x}}{\alpha^2+\beta^2}(\alpha\sin(\beta x)+\beta\cos(\beta x))+C$$

Therefore, $\frac{S_{n+1}}{S_n}=e^{\frac{-\alpha\pi}{\beta}}$

3.(D) The expected value $E(N)=\sum kp_k$, where *k* is the number of fixed points and p_k is the probability of having *k* fixed points. Notice that:

$$p_k=\binom{n}{k}\frac{D_{n-k}}{n!}=\binom{n}{n-k}\frac{D_{n-k}}{n!}$$

Where $D_m=$ derangement of *m* things

$$E(n)=\sum_{k=0}k\binom{n}{k}\frac{D_{n-k}}{n!}=\sum_{m=0}^{n}(n-m)\binom{n}{m}\frac{D_m}{n!}$$

$$E(n)=\sum_{m=1}^{n}(n-m)\binom{n}{m}\frac{D_m}{n!}=\sum_{m=1}^{n}\frac{D_m}{m!(n-m-1)!}$$

Multiplying both sides by $(n-1)!$, we get:

$$(n-1)!E(n)=\sum_{m=1}^{n}\binom{n-1}{m}D_m$$

But it can be easily seen that $\sum_{m=1}^{n}\binom{n-1}{m}D_m$ is equal to $(n-1)!$ because every permutation has a certain number of fixed points with remaining points deranged.

$\Rightarrow \quad (n-1)!E(n)=(n-1)! \;\Rightarrow\quad E(n)=1,$

For all *n*.

4.(A) Use induction to prove that $z_k=i^k\binom{n}{k}z_0$ for all $k\in\{0,1,2,...,n-1\}$.

Then, $z_0+z_1+z_2+...+z_n=2^n$ if and only if

$$z_0(1+i)^n=2^n \qquad\Rightarrow\qquad z_0=(1-i)^n.$$

$$|z_0|^2+|z_1|^2+|z_2|^2+...+|z_n|^2=|z_0|^2\left(\binom{n}{0}^2+\binom{n}{1}^2+\binom{n}{2}^2+...+\binom{n}{n}^2\right)$$

$$=|z_0|^2\binom{2n}{n}=2^n\binom{2n}{n}=\frac{2^n}{n!}2n(2n-1)...(n+1)<\frac{2^n}{n!}\left(\frac{2n+(2n-1)+...+(n+1)}{n}\right)^n$$

Also, by applying AM-GM inequality, we have $=\dfrac{(3n+1)^n}{n!}$.

SECTION-2

5.(AD)

If the grasshopper stands at (0, 0) and goes to a point (*x*, *y*), then $x^2+y^2=25$ and *x* and *y* must be integers. So $x=\pm3$ or $x=\pm4$ and $y=\pm4$ or $y=\pm3$. OR $x=\pm5, y=0$ and vice-versa So, only twelve movements are allowed. Each hop corresponds to adding one of the 12 vectors $(0,\pm5),(\pm5,0),(\pm3,\pm4),(\pm4,\pm3)$ to the position of the grasshopper. So using (3, 4) and (4, 3) it can reach (7, 7). Since (2021, 2021) = 288(3, 4) + 288(4, 3) + (0, 5) + (5, 0) , the grasshopper can reach (2021, 2021) in 288 + 288 + 1 + 1 = 578 hops. On the other hand, let *z* = *x* + *y* denote the sum of the x and y coordinates of the grasshopper, so that it starts at *z* = 0 and ends at z = 4042. Each hop changes the sum of the x and y coordinates of the grasshopper by at most 7, and

4042 > 577x7; it follows immediately that the grasshopper must take more than 577 hops to get from (0, 0) to (2021, 2021).

6.(ABD)

$g : R \to \{4\}$

$g(x) = x^3\left[f'(t) - 2\right] + x^2 f''(t) + 4x\left[f(0) + 6\right] + 4 = 4$

$\therefore \quad x\left[x^2(f'(t) - 2) + x(f''(t)) + 4(f(0) + 6)\right] = 0 \qquad \forall x \in R$

$\therefore \quad f'(t) = 2 \quad \Rightarrow \quad f(t) = 2t + c$

$f''(t) = 0 \quad \& \quad f(0) = 0 + c = -6$

$f(0) = -6 \qquad c = -6$

So $\to \quad f(t) = 2t - 6$

$$h(x) = \begin{cases} \int_0^x |2t-8|\,dt, & 0 \le x \le 6 \\ (x-6)^2 + 20, & 6 < x \le 12 \end{cases}$$

$$h(3) = \int_0^3 |2t-8|\,dt = \int_0^3 (8-2t)dt = [8t - t^2]_0^3 = 24 - 9 = 15$$

$$h(6) = \int_0^6 |2t-8|\,dt = \int_0^4 (8-2t)dt + \int_4^6 (2t-8)dt$$

$$= [8t - t^2]_0^4 + [t^2 - 8t]_4^6 = 32 - 16 + 36 - 48 - 16 + 32 = 16 - 4 = 20$$

$$h(12) = (12-6)^2 + 20 = 36 + 20 = 56$$

$\therefore \quad$ Range of $h(x) = [0, 56]$

7.(CD)

$$P_2 = P(T) + P(HH) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$P_3 = P(TH) + P(HT) + P(HHH) = \frac{1}{4} + \frac{1}{4} + \frac{1}{8} = \frac{5}{8}$$

$$P_n = P_{n-1} \times P(H) + P_{n-2}P(T)$$

$$P_n = \frac{1}{2}\left(P_{n-1} + P_{n-2}\right)$$

$$P_4 = \frac{1}{2}\left[P_3 + P_2\right] = \frac{1}{2}\left[\frac{5}{8} + \frac{6}{8}\right] = \frac{11}{16}$$

8.(ABC)

By AM − GM inequality, we have:

$n + (n+1) > 2\sqrt{n(n+1)} \qquad \Rightarrow \qquad 2(n+1) - 2\sqrt{n(n+1)} > 1$

$$\Rightarrow \qquad \frac{1}{\sqrt{n}(n+1)} < \frac{2}{\sqrt{n}} - \frac{2}{\sqrt{n+1}}$$

$$\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}(n+1)}\right) < \sum_{n=1}^{\infty}\left(\frac{2}{\sqrt{n}} - \frac{2}{\sqrt{n+1}}\right) = 2$$

9.(AD) The given recurrence is: $a_{n+1} = a_n + \frac{1}{a_n^2}$

Cubing this recurrence, we get $a_{n+1}^3 = a_n^3 + 3 + 3\frac{1}{a_n^3} + \frac{1}{a_n^6}$.

So, we get: $a_{n+1}^3 > a_n^3 + 3$. Inductively $a_n > \sqrt[3]{3n}$, which gives us all the four options.

10.(AC)

WLOG $a = 1, b = e^{i\theta}, c = e^{i\phi}$.

$|a+b+c| \le 1 \quad \Rightarrow (a+b+c)(\bar{a}+\bar{b}+\bar{c}) \le 1.$

$(1+e^{i\phi}+e^{i\theta})(1+e^{-i\phi}+e^{-i\theta}) \le 1 \quad \Rightarrow \quad 4\left(\cos\frac{\theta}{2}\right)^2\left(\cos\frac{\phi}{2}\right)^2 + \sin\theta\sin\phi \le 0$

$\Rightarrow \qquad \sin\theta\sin\phi \le 0$

Consider: $\qquad |a^2+bc|^2 = (a^2+bc)(\bar{a}^2+\bar{b}\bar{c}) \ge (b+c)(\bar{b}+\bar{c})$

$(1+e^{i(\theta+\phi)})(1+e^{-i(\theta+\phi)}) \ge (e^{i\theta}+e^{i\phi})(e^{-i\theta}+e^{-i\phi})$

Which is the same as: $\sin\theta\sin\phi \le 0$.

Similarly for the other part.

SECTION-3

1-2. **1.(0.33) - 2.(4)**

There are 16 equally probable cases out of which only 3 are feasible in the given condition.

(a) A book in box numbered 1 and a book in box numbered 4

(b) A book in box numbered 2 and a book in box numbered 4

(c) A book in box numbered 3

So, the probability of box numbered 3 containing a book is $\frac{1}{3}$ and as it can be seen that box numbered 4 has the largest probability of containing the book.

3-4. **3.(1) - 4.(9)**

If P divides BC in the ratio $\lambda : 1$, then the coordinates of P are

$$\left(\frac{\lambda x_3 + x_2}{\lambda+1}, \frac{\lambda y_3 + y_2}{\lambda+1}\right)$$

Also, as P lies on L, we have

$$l\left(\frac{\lambda x_3 + x_2}{\lambda+1}\right) + m\left(\frac{\lambda y_3 + y_2}{\lambda+1}\right) + n = 0$$

$$\Rightarrow \qquad -\frac{lx_2 + my_2 + n}{lx_3 + my_3 + n} = \lambda = \frac{BP}{PC} \qquad \ldots(1)$$

Similarly, we obtain

$$\frac{CQ}{QA} = -\frac{lx_3 + my_3 + n}{lx_1 + my_1 + n} \qquad \ldots(2)$$

and $\frac{AR}{RB} = -\frac{lx_1 + my_1 + n}{lx_2 + my_2 + n} \qquad \ldots(3)$

Multiplying (1), (2) and (3) we get

$$\frac{BP}{PC}\times\frac{CQ}{QA}\times\frac{AR}{RB}=-1$$

In $\frac{x_1+x_2+x_3}{3}=0,\ \frac{y_1+y_2+y_3}{3}=0$ ...(4)

And $\frac{lx_1+my_1+n}{\sqrt{l^2+m^2}}+\frac{lx_2+my_2+n}{\sqrt{l^2+m^2}}+\frac{lx_3+my_3+n}{\sqrt{l^2+m^2}}=1$

$\Rightarrow \quad l(x_1+x_2+x_3)+m(y_1+y_2+y_3)+3n=\sqrt{l^2+m^2}$

$\Rightarrow \quad l^2+m^2=9n^2$ (from (4))

Sum of the squares of the reciprocal of intercepts made by L on the coordinate axis is $\left(\frac{l}{-n}\right)^2+\left(\frac{m}{-n}\right)^2=\frac{l^2+m^2}{n^2}=9.$

5.(2) Equation of tangent to parabola $4ay=x^2$ of slope m is

$y=mx-am^2$

$y=mx-\frac{m^2}{8}$

$m^2-8mx+8y=0$

$(p\ p+1)\ \Rightarrow\ m^2-8mp+8(p+1)=0$

for distinct tangents $D>0$

$64p^2-32(p+1)>0$

$p\in\left(-\infty,-\frac{1}{2}\right)\cup(1,\infty)$

6.(16) The tangent is:

$y=mx-\frac{m^2}{4a}.$

$y=mx-\frac{m^2}{8}$

$K+3=Km-\frac{m^2}{8}$

$K=2$ Sum of $=16$

SECTION-4

7.(3) Let $g(x)=x^2-2mx-4(m^2+1);\ D_1=4(5m^2+4)$

$h(x)=x^2-4x-2m(m^2+1);\ D_2=8(2+m^3+m)$

For $f(x)$ to have exactly three distinct linear factors, $g(x)$ & $h(x)$ must have total 3 distinct zeroes.

Case-I $\Rightarrow\ D_2=0\ \Rightarrow\ m^3+m+2=0 \quad \Rightarrow\ (m+1)(m^2-m+2)=0\ \Rightarrow\ m=-1$

For $m=-1$

$h(x)=x^2-4x+4\ \Rightarrow\ (x-2)^2$

$g(x)=x^2+2x-8\ \Rightarrow\ (x+4)(x-2)$

Both have total two zeros – 4, 2

Case-II $\Rightarrow$ If $g(x), h(x)$ have exactly one common root (let α)

$$g(\alpha)-h(\alpha)=\{\alpha^2-2m\alpha-4m^2-4\}-(\alpha^2-4\alpha-2m^3-2m)=0$$

$$\Rightarrow \quad 2(2-m)\alpha+2m^2(m-2)+2(m-2)=0$$

$$\Rightarrow \quad 2(2-m)\{\alpha-m^2-1\}=0 \Rightarrow \alpha=m^2+1 \text{ or } m=2$$

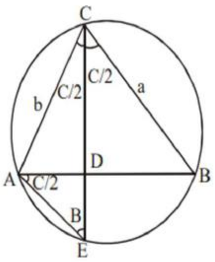
$$f(\alpha)=0 \Rightarrow f(m^2+1)=0 \quad \Rightarrow \quad (m^2+1)^2-2m(m^2+1)-4(m^2+1)=0$$

$$(m^2+1)\{m^2+1-2m-4\}=0$$

$$(m^2+1)(m^2-2m-3)=0 \quad \Rightarrow (m^2+1)(m-3)(m+1)=0$$

Hence finally *m* can be 3 only as for *m* = 2, both roots are common.

8.(3) $\angle BAE=\dfrac{C}{2}$



as BE subtends equal angles at C and A

In $\Delta ACD,$ by sine rule $\dfrac{CD}{\sin A}=\dfrac{AD}{\sin(C/2)}$

In $\Delta ADE,$ $\dfrac{AD}{\sin B}=\dfrac{DE}{\sin(C/2)}$

Equating *AD*, we get,

$$\frac{CD.\sin(C/2)}{\sin A}=\frac{DE.\sin B}{\sin(C/2)}$$

$$\therefore \quad \frac{CD}{DE}=\frac{\sin A\sin B}{\sin^2(C/2)}$$

Add 1,

$$\frac{CD}{DE}+1=\frac{\sin A\sin B+\sin^2(C/2)}{\sin^2(C/2)} \quad \Rightarrow \quad \frac{CE}{DE}=\frac{2\sin A\sin B+2\sin^2(C/2)}{2\sin^2(C/2)}$$

$$=\frac{\cos(A-B)-\cos(A+B)+1-\cos C}{2\sin^2(C/2)}$$

$$=\frac{1+\cos(A-B)}{2\sin^2(C/2)} \qquad \{\because \cos(A+B)=\cos(\pi-C)=-\cos C\}$$

$$=\frac{2\cos^2\left(\frac{A-B}{2}\right)}{2\sin^2(C/2)}=\left[\frac{\cos\left(\frac{A-B}{2}\right)}{\sin\left(\frac{C}{2}\right)}\right]^2=\left[\frac{2\cos\left(\frac{A-B}{2}\right)\cos\left(\frac{C}{2}\right)}{2\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)}\right]^2$$

$$=\left(\frac{\cos\left(\frac{A-B+C}{2}\right)+\cos\left(\frac{A-B-C}{2}\right)}{\sin C}\right)^2=\left(\frac{\sin B+\sin A}{\sin C}\right)^2$$

$$=\left(\frac{b+a}{c}\right)^2 \qquad \{\text{using sine rule}\}$$

$\therefore \quad k=1;\quad p=2\Rightarrow k+p=3.$

9.(4) $\vec{a}\times\vec{b}=p\vec{a}+q\vec{b}+r\vec{c}$

Taking dot product with $\vec{a},\vec{b},\vec{c}$

$$0=p+\frac{q}{3}+\frac{r}{3} \qquad \ldots(1)$$

$$0=\frac{p}{3}+q+\frac{r}{3} \qquad \ldots(2)$$

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\frac{p}{3}+\frac{q}{3}+r \qquad \ldots(3)$$

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2=\begin{vmatrix}1&\frac{1}{3}&\frac{1}{3}\\\frac{1}{3}&1&\frac{1}{3}\\\frac{1}{3}&\frac{1}{3}&1\end{vmatrix}=1\left(1-\frac{1}{9}\right)+\frac{1}{3}\left(\frac{1}{9}-\frac{1}{3}\right)+\frac{1}{3}\left(\frac{1}{9}-\frac{1}{3}\right)=\frac{8}{9}-\frac{2}{27}-\frac{2}{27}=\frac{8}{9}-\frac{4}{27}=\frac{20}{27}$$

From (1) and (2), we get $p=q, r=-4q$

From (3) $\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\frac{q}{3}+\frac{q}{3}-4q=-\frac{10q}{3}$

$$\frac{20}{27}=\frac{100}{9}q^2 \quad\Rightarrow\quad q^2=\frac{1}{15}$$